Lecture 13. **Synthesis of the linear ACS. Stages of the solving of the task of parametric synthesis of the linear ACS**

The first problem which had been being solved in the theory of automatic control was to provide stability of dynamic systems. Stability is a necessary but insufficient property of an automatic control system. Sufficiency of automatic control system (ACS) allows achieving the required quality of control. Systematization and summarizing of the information collected had let to develop further methods of task solution of ACS synthesis with target quality indexes.

This chapter describes objectives and methods of ACS analysis. Then hereby the task itself and the methods of ACS parametric synthesis with target quality indexes are set in the terms "input-output" and in states space.

But, before starting to solve the task of ACS parametric synthesis, it is necessary to check if the dynamic system under consideration is controlled and observed? For this in present chapter we examine criteria of controllability and observability by R. Kalman and E. Gilbert.

13.1 Objectives and methods of ACS analysis. Task of ACS synthesis setting

The problem of providing required properties for linear automated systems is to be highly complicated. First of all, it consists of the following objectives: to provide stability; to increase stability factor (damping); to increase control accuracy in steady states (decrease or elimination of steady-state replication error for master control); to improve transient processes (speedup, decrease of dynamic error from disturbances).

Generally, objectives of ACS analysis are divided into three main sections: stability analysis, evaluation of proper movement, evaluation of control quality.

1) Analysis of ACS stability

Any automated control system should be stable. Analysis of ACS stability is carried out with the help of criteria of stability, which are described in details in chapter 3 of the present book.

If mathematical formulation of the system is set in terms "input-output", then analysis of stability will be carried out by algebraic and frequency criteria. If the order of the system is not high ($n \leq 4$), then it is advisable to apply algebraic criterion of stability by A. Hurwitz; at $n > 4$ it is advisable to apply criterion of stability by E. Routh. Criterion of stability by A.V. Mikhailov is advisable to apply when investigating complicated multicircuit systems in frequency sphere; criterion by Yu. Nyquist is reasonable when investigating complicated multicircuit systems with delay or when characteristics of links are set experimentally.

If mathematical formulation of dynamic system is set in state space, then analysis of the system stability should be carried out with the help of A.M. Lyapunov's theorems described in chapter 3 of the present book.

But, quite often we have to investigate the impact of one or two parameters of the system on its stability. In chapter 3 of the present book space *D*-partition of parameters method, proposed by Yu. I. Neumark is described in more details.

2) Estimation of free movements of the system

Analysis of proper movements of the system or root method and making a transient function as the main feature of dynamic system is discussed here.

Determination of principle evaluations of proper movements of the system is performed by:

– the type of transient function: aftereffect of the system, oscillation, coefficient of steady-state error;

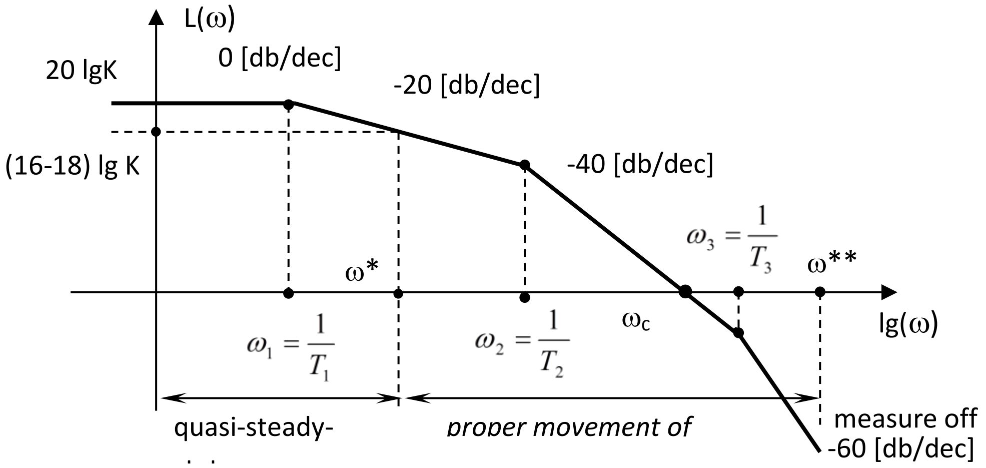
– integral criteria of transient functions; integral of transient function, integral of transient function square.

In this section we should mark out objectives of analysis, solved by root and frequency methods. In this case it is necessary to remember that frequency characteristics always define characteristics of steady-state regime. The unique dependence between aftereffect and cutoff frequency or bandwidth (main feature of generalized frequency function) is observed; if LgGFC (Fig. 5.1) has an incline *-40 [db/dec]* in point $L(\omega)=0$, it shows an evidence of oscillation of the process, etc.

Pattern of transient process is a more complete characteristics of dynamic system, as this is a characteristics of transition of the system from one state to an other one.

3) Estimation of control quality

The principle measure of determining quality of the system is the error of control. Error coefficients are determined by factoring the error to the row. On other hand, error coefficients may be defined by frequency parameters of dynamic system. On fig. 5.1 the LgGFC of open-loop system is presented where relevant regimes are marked.



Pic. 5.1. LgGFC of open system

Here ω_{cp} is cutoff frequency or bandwidth; from ω^* to ω^{**} is interval where proper movement of the system is determined; $\omega^* \approx$ *(16-18) lg K*, where *"K"* is determined from reasonable steady error.

If at frequencies that are greater than ω^{**} the signal depression in output is carried out to an order, that means that this signal may be neglected as well as other unaccounted factors. This is a measure of evaluation crudity of the system investigated. Generally speaking, characteristic numbers of the system are located from $-\infty$ to $+\infty$, but after ω^{**} characteristic numbers are negligible.

If we increase amplification coefficient, the line with an incline *–20 [db/dec]* will be descended and cutoff frequency ω_c will be shifted forward. This means that system speed is increasing; shift of ω^* and ω^{**} widens the area of essential definition of system parameters. Therefore, the area of negligible values of time constants $W(s)=\dfrac{Q_1(s)}{Q_2(s)}$ is decreasing. Some new roots are introducing, mathematical formulation is adjusting, and the order of differential equation is increasing.

Objective of analysis as definition of principle measures of proper movement evaluation and error characteristics when adjusting controlling impacts is solved only after when the following is set:

a) characteristics of dynamic system;

b) structure of the system, that is principle of control and the adjustment law.

13.2 Tasks of Automatic Control Systems synthesis. Synthesis problem definition

The term "task of ACS synthesis" is interpreted as a task of determination of the principle system parameters, which characterize the selected regulations of control and which satisfy to all prescribed requirements:

a) on characteristic features of stability of the system;

b) on characteristic features of errors appearing in steady-state regime.

These two requirements (a) and (b) are internally a little bit contradictory. Requirements to ACS may be formulated in two ways: to transient process *h(t);* to the error is $W_\varepsilon(s)=\dfrac{\varepsilon(s)}{Q_{in}(s)}$ in a steady-state mode.

If the requirements had been formulated to the error, this task is called *the task of design of the system.*

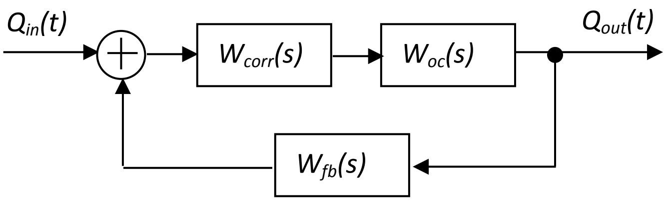


Fig. 5.2a. Closed-loop system with a correcting device

At specified CO parameters, dynamic properties of which essentially and fundamentally cannot be changed, it is necessary by using the selected way of control on mismatching, to introduce such corrective link $W_{corr}(s)$, in order the error of control to be no more than the specified one.

In this case the task of synthesis is to be interpreted as determination of links parameters, introduced into a forward path of input signal and into feedback path.

The task of synthesis as an impose of definite requirements to the quality of transient process *h(t)* at specified way of control is called *the parametric synthesis task.*

It is always necessary to strive to that the transfer function of closed-loop system was identically equal to one:

$$W_{closed}(s) \overset{\Delta}{=} \frac{Q_{out}(s)}{Q_{in}(s)} = \frac{Q_1(s)}{Q_2(s)} \equiv 1 \quad \text{(at any numerator } Q_2(s)\text{)}.$$

The main properties are specified with OC characteristics, and it is admissible to impact on characteristic numbers in the closed-loop system only in limited scale or *in extremely realized ones*.

Does the CO reaction time depend on input signal? Sure, it does. At unlimited input action of control we may achieve any type of speeding. But practically it is impossible to do. Speeding requirements should comply with the power of executive component of regulator.

It is not so difficult to determine control. Accuracy or a measure of quality evaluation is defined by a priori knowledge of CO characteristic features. If CO properties are changed, then we need to keep track current state of CO and warn the control. This caused the appearance of adaptive control.

13.3 Stages of solution of parametric synthesis task

1) To specify CO characteristics, select this or that CO mathematical model or in the terms “input-output” as a transfer function

$$W(s) = \frac{Q_{out}(s)}{Q_{in}(s)}$$

or in state space

$$\begin{cases} \dot{X} = AX + BU \\ Y = CX \end{cases}.$$

2) To check the original system on stability at specified way of control on mismatch.

3) To specify the requirements to quality of transient process *h(t)*: speeding t_p, percentage of overcontrol %σ, static accuracy σ, etc.

4) To define parameters of transfer function of correcting device in forward or feedback path.

5) To make a transient process *h(t)* the main characteristic of the automated control system; characterization of quality performances.

As a result we obtain the contradiction between received characteristics of transient process and desired ones. This is because of some non-compliance between CO mathematical model of uncontrolled part of the system and requirements to quality of transient process.

Hence there is a next stage:

6) correcting process of:

– CO mathematical model of uncontrolled part of the system, i.e. account of neglected roots of characteristic equation;

– ACS regulation of control, i.e. account of the 1^{st} and 2^{nd} derivative in regulation of control;

– changing the way of control.

Therefore, we obtained an iterative procedure of solution of ACS parametric synthesis task, consisting six steps. If it is converged, then the parametric synthesis task is solved. If it varies, then the requirements to transient process do not comply with abilities of CO mathematical model.